The Origins of Chaos Theory

-Brian Capozzi

In his work, The Origin of Wealth, Eric Beinhocker discusses chaos and its applications to complexity economics. Chaos theory has provided a fascinating new lens through which the world can be viewed. Prior to the work of such pioneers as Henri Poincare and Edward Lorenz, the universe was mainly studied in terms of linear systems: systems in which small changes have small consequences. These linear systems are predictable with high degrees of accuracy, and it was believed that in any system which exhibited unpredictability, that this unpredictability could be attributed to random external forces. Chaos theory has expelled this thought approach, and it has arisen to explain the behaviors of nonlinear, dynamic systems. These are systems which are often difficult, if not impossible, to predict. Yet, in order for a system to be called chaotic, there are to be no random elements within that system. So, if a chaotic system doesn't exhibit randomness, what exactly does chaos theory entail, and from where did it originate?

Late in the 1800's, Henri Poincare was one of the first individuals to address the phenomenon of chaos in his attempt to solve the three body problem. In classical mechanics, a 3-body problem is one in which, given a system of three bodies with initial positions, masses, and velocities, the future motions of the system are attempted to be calculated (this is a specific case of the n-body problem: one which contains 'n' number of masses) (Davies, pg 6). The prediction of a two-body system is both a deterministic and a feasible calculation. Yet, when a third body is thrown into the mix, the system is thrown into chaos. This threshold is exactly like the same idea of the example given by Beinhocker when he discusses the "edge of chaos." In his example, when one light bulb was connected to four others in a Boolean switching network, the behavior of the bulbs was thrown into chaos. In the case of the n-body problem, the threshold for chaos is two bodies; anything more and the system becomes highly unpredictable as it exhibits chaotic behavior.

Poincare's work on the subject was submitted as an entry to an international competition held in honor of King Oscar II's 60th birthday, and it was later published in 1890. He won the competition— along with its monetary prize— although he was unable to actually solve the stated problem (Davies, pg 7). Instead, Poincare's discoveries and postulations led to a new way of analyzing nonlinear, dynamic systems and essentially introduced the study of chaos. Poincare focused on a specific case of the three-body problem: a case in which one of the three bodies has a mass that is negligible in comparison to the other two masses (this removes it's gravitational influence on the other two masses). He discovered that periodicity would not necessarily occur in this situation, and that small differences in initial set-ups led to vastly different future system states (Davies, pg 7). He also (insightfully) pointed out that due to the chaotic nature of an n-body problem, that the very structure of our solar system was not guaranteed. These ideas were paramount in the founding of chaos theory.

However, while Poincare's work was groundbreaking, the field of chaos theory was unable to truly take off until the invention of the computer. Analyzing chaotic systems requires the repetitive solution of equations governing the systems' behavior; done by hand this is incredibly tedious, time-consuming, and impractical. Edward Lorenz, an American meteorologist, made strides in the field of chaos theory during the 1960's while attempting to use computer models to forecast the weather. To his surprise, while running a simulation with what he thought were the same parameters from a prior simulation, he obtained a very different conclusion (Davies, pg 10). In fact, what Lorenz thought were the same parameters were actually values that had been slightly truncated on a printout; under current theory, this minor change in value should not have had any profound effect on the outcome of his predictions. Yet, these extremely minor differences led to drastically different weather forecasts.

The differential equations considered by Lorenz:

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = xy - bz$$

are a simplification of equations used to model convection currents driven by a heat source from below. In these equations, x, y, and z represent state variables, while σ, r, and b represent constants which depend upon the nature of the system. While these equations may not seem very intimidating, they allowed Lorenz to observe the nature of chaotic motion and refuel the field of chaos theory. Their solutions yield results which are incredibly sensitive to initial conditions and which exhibit non-periodic behavior: two hallmarks of chaos. When Lorenz spoke on his findings, he popularized the term "butterfly effect" in the talk's title: "Predictability: Does the flap of a Butterfly's Wings in Brazil set off a Tornado in Texas?" (Davies pg 10) While the talk's title is clearly an exaggeration, it encapsulates one of the key aspects of chaos theory: tiny changes having large consequences.

Now that the foundations of chaos theory have been described, what exactly is chaos theory? Chaos theory is more like a tool which seeks to understand the dynamics of systems which exhibit seemingly unpredictable actions. Such systems analyzed by chaos theory are— as has been shown in the examples of Poincare and Lorenz— extremely sensitive to initial conditions. While chaotic systems may at first appear to be completely random, it is important to note that there is no element of randomness in truly chaotic behavior; chaotic stystems are actually deterministic and adhere to strict mathematical formulas which govern their dynamics.

While chaotic systems can be modeled, the longer the time period of the model, the less accurate the results are. This occurs because of the aforementioned butterfly effect in which a tiny discrepancy in an initial condition can be greatly amplified and have far-flung future results. In other words, initial data which isn't or can't be measured to a fine enough precision will greatly impact the evolution of that system. Effects such as these can be clearly seen in weather forecasting where it is currently impossible to accurately predict the weather past one week. In fact, Lorenz himself believed that it would never be possible to exceed this accuracy because there are just too many variables which would have to be extremely precisely taken into account. A final, important, aspect of chaos theory is that chaotic systems do not exhibit periodicity; no matter how long the system runs it will never perfectly replicate a pattern. This is a key point because there are many systems in which periodic behavior is masked by elaborate and long periods of activity.

Chaos theory has become so successful that it has gained recognition and application as a tool for the analysis of many real world situations. Many of these situations were previously thought to be completely random, but they now receive analysis under chaos theory. It plays important roles in the understanding of turbulence (in fluid mechanics), entropy being released in thermodynamic processes, fluctuations in lasers, and (as previously discussed) forecasting the weather. It is also now being used to gain a further understanding of biological processes such as evolution, as well as heart arrhythmias and brain function. Furthermore, chaos theory concepts have been used as a basis for programming events into video games. (Charap pgs 64-67)

Chaos theory has introduced an entirely new way of studying the world. Thanks to pioneering efforts by Poincare and later by Lorenz, the world is no longer seen in terms of linear systems and systems which have random exogenous forces not believed to be taken into account by models. Instead, there is a whole realm of systems—chaotic systems— in which predictability is no longer viable because of such critical dependence on initial conditions. While these chaotic systems *could* be modeled, many (such as the weather) cannot because it is not feasible with modern day technology to measure variables with enough accuracy. Chaos theory continues to expand its horizons as it is applied to new fields; hopefully it will continue to provide new insights into natural phenomena.

Works Cited

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